

Dynamic simulation of a beta-type Stirling engine with cam-drive mechanism via the combination of the thermodynamic and dynamic models

Chin-Hsiang Cheng*, Ying-Ju Yu

Department of Aeronautics and Astronautics, National Cheng Kung University, No. 1, Ta-Shieh Road, Tainan, Taiwan 70101, R.O.C

ARTICLE INFO

Article history:

Received 6 April 2010

Accepted 28 July 2010

Available online 6 September 2010

Keywords:

Dynamic simulation

Stirling engine

Beta type

Thermodynamic and dynamic models

ABSTRACT

Dynamic simulation of a beta-type Stirling engine with cam-drive mechanism used in concentrating solar power system has been performed. A dynamic model of the mechanism is developed and then incorporated with the thermodynamic model so as to predict the transient behavior of the engine in the hot-start period. In this study, the engine is started from an initial rotational speed. The torques exerted by the flywheel of the engine at any time instant can be calculated by the dynamic model as long as the gas pressures in the chambers, the mass inertia, the friction force, and the external load have been evaluated. The instantaneous rotation speed of the engine is then determined by integration of the equation of rotational motion with respect to time, which in return affects the instantaneous variations in pressure and other thermodynamic properties of the gas inside the chambers. Therefore, the transient variations in gas properties inside the engine chambers and the dynamic behavior of the engine mechanism should be handled simultaneously via the coupling of the thermodynamic and dynamic models. An extensive parametric study of the effects of different operating and geometrical parameters has been performed, and results regarding the effects of mass moment of inertia of the flywheel, initial rotational speed, initial charged pressure, heat source temperature, phase angle, gap size, displacer length, and piston stroke on the engine transient behavior are investigated.

© 2010 Elsevier Ltd. All rights reserved.

1. Introduction

Among the most related solar energy technologies, concentrating solar power systems with Stirling engines (CSP-Stirling) have been treated as one of the most promising energy sources in recent years. The CSP-Stirling systems require solar absorbers to receive solar radiation and then use the Stirling engines to convert the received solar energy to electrical power. Theoretically, the performance of the systems for the generation of electrical power is dependent on the receiver temperature and the thermal efficiency of the engines. It has been demonstrated that the highest system efficiency of the CSP-Stirling system reaches 30% in converting direct-normal incident solar radiation into electricity after accounting the power losses [1]. The SES 25-kW SunCatcher [2] consisting of a solar dish concentrator, a solar tracker, and a Stirling engine has received great attention from the energy-related researchers in recently. In the work of Reinalter et al. [3], the performance of a 10-kW dish/Stirling system was evaluated, which was considered to be the latest development step of the EuroDish system by improving many components. An axisymmetric

computational fluid dynamics approach to the analysis of the working process of a solar Stirling engine was performed by Mahkamov [4]. Liao [5] also conducted a simulation of the CSP system and assessed its feasibility in the Taiwan area. Most recently, a number of new articles [6–8] have come to attention regarding the CSP technology.

In the past several decades, numerous researches [9–13] have been performed for the development of the Stirling engines. A detailed literature review for the solar-powered Stirling engines was given by Kongtragool and Wongwises [14].

Modeling of the physical transport phenomena in the Stirling engines has also received great attention in the past several decades. The Schmidt theory [15] has been used widely due to its mathematical simplicity. The non-isothermal analysis was first carried out by Finkelstein in 1960s [16]. The author evaluated the heat transfer coefficient and obtained the heat transfer and the temperature variation in the working space. This work was notified as one of the major improvement in the modeling of Stirling engine processes. Lately, more analytical and simulation models have been proposed [17–22] for predicting the performance of the Stirling engines.

By using a quasi-steady flow model based on the works of Urieli and Berchowitz [20], thermodynamic analysis of a gamma-type

* Corresponding author. Tel.: +886 6 2757575X63627; fax: +886 6 2389940.
E-mail address: chcheng@mail.ncku.edu.tw (C.-H. Cheng).

Nomenclature

a	Acceleration (m/s^2)
C_v	Constant volume specific heat (kJ/kg K)
C_p	Constant pressure specific heat (kJ/kg K)
dt	Time step (s)
G	Regenerative channel gap (m)
h	Enthalpy (kJ/kg)
I	Mass moment of inertia (kg m^2)
l_d	Length of displacer (m)
l_p	Length of piston (m)
l_t	Height of the engine (m)
l_1, l_2	Lengths of the link 2–3 and 5–6 of the cam-drive mechanism (m)
l_{dt}	Length from the joint 5 to the lower surface of displacer (m)
m	Mass (kg)
$\dot{m}$	Mass flow rate (kg/s)
P	Pressure (kPa)
Q	Volumetric flow rate (m^3/s)
r_1	Radius of displacer (m)
r_2	Core radius of cylinder (m)
R	Gas constant (J/kg K)
R_d	Offset distance from the displacer crank to the center of gear (m)
R_p	Offset distance from the piston crank to the center of gear (m)
R_{t1}	Thermal resistance of heating head ($^\circ\text{C/kW}$)
R_{t2}	Thermal resistance of cooling jacket ($^\circ\text{C/kW}$)
t	Time (s)
t_p	Period (s)
T	Temperature (K)
T_H	Heat source temperature (K)
T_L	Heat sink temperature (K)
U	Internal energy (kJ/s)
v	Velocity (m/s)

V	Volume (m^3)
W	Weight of the components (Kg)
W_1	Weight of link 2–3 (Kg)
W_2	Weight of link 5–6 (Kg)
$\bar{W}$	Cycle-average output power (W)
Y	Displacement (m)

Greek symbols

β_1, β_2	Angle between the central axis and the linkages 2–3 and 5–6 (rad)
η	Thermal efficiency
μ	Dynamic viscosity (kg/m s)
θ	Crank angle (rad)
ρ	Fluid density (kg/m^3)
ω	Rotational speed (rad/s)
α	Angular acceleration (rad/s^2)
ε	Rod's angular acceleration (rad/s^2)
ε_{eff}	Regeneration effectiveness
Φ	Phase angle (degree)

Subscripts

ave	Cycle-average
c	Compression chamber
d	Displacer
e	Expansion chamber
eff	Effectiveness
fr	Friction
i	Initial value
load	External load
p	Piston
r	Regenerative channel
s	Steady state
tin	Tangential force
2–3	link 2–3
5–6	link 5–6

Stirling engine in non-ideal adiabatic conditions was presented by Parlak et al. [21]. In the work of Karabulut et al. [22], a beta-type Stirling engine was built and tested. The beta-type Stirling engine was considered to have the largest specific cyclic work as compared to other types of the Stirling engines. Most recently, Cheng and Yu [23] presented a numerical model for a steadily-operated beta-type Stirling engine with rhombic-drive mechanism. By taking into account the non-isothermal effects, the effectiveness of the regenerative channel, and the thermal resistance of the heating head, the authors carried out the parametric study of the dependence of the output power and thermal efficiency on the geometrical and physical parameters, involving regenerative gap, distance between two gears, offset distance from the crank to the center of gear, and the heat source temperature.

However, in practices, the study of the transient operating process of the engines is essential and necessary, and hence dynamic simulation of the engine forms another critical issue of the engine's operation performance. Unfortunately, to the authors' knowledge, most existing simulation model for the beta-type Stirling engines analysis only dealt with the steady-state performance of the engines, and the transient behavior of the engine in the start period has not been sufficiently investigated.

For an internal combustion engine, Giakoumis and Rakopoulos [24] provided a numerical model accounting for the transient conditions with the slider-crank-driving mechanism. Lately, the same group of authors [25] extended the study to include the

instantaneous torsional deformation in both the steady-state and the transient conditions. The crankshaft angular momentum equilibrium was formulated by taking into account instantaneous gas, inertia and friction loads as well as the stiffness and damping torque contributions. However, the model for the internal combustion engines developed in Refs. [24,25] cannot be directly applied for the beta-type Stirling engines.

Under these circumstances, to predict the transient behavior of the beta-type Stirling engines with cam-drive mechanism during the hot-start period, in the present study a dynamic model is developed and then incorporated with the thermodynamic model developed by Cheng and Yu in [23] so as to investigate the transient variations in gas properties inside the engine chambers and the dynamic behavior of the engine mechanism. By combining the thermodynamic model [23] and the present dynamic model, an extensive parametric study of the effects of mass moment of inertia of the flywheel, initial rotational speed, initial charged pressure, heat source temperature, phase angle, gap size, displacer length, and piston stroke on the engine's transient behavior is performed, and results coming out will be discussed in the subsequent sections.

2. Dynamic model

The schematic diagram of the beta-type Stirling engines with cam-drive mechanism is illustrated in Fig. 1. The concentric displacer and the piston are placed in one single cylinder. A connecting

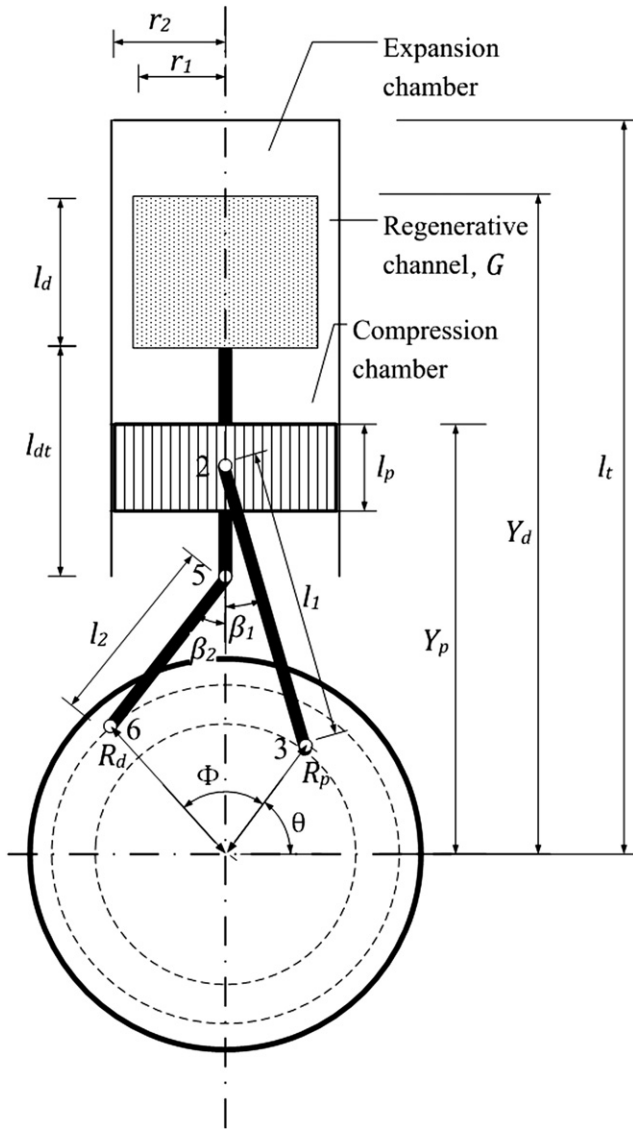


Fig. 1. Schematic diagram of the beta-type Stirling engine with cam-drive mechanism.

rod connects the displacer and the flywheel. The joint between the rod and the flywheel is distant from the flywheel center at an offset distance, R_d . Another rod connects the piston and the flywheel, and the joint between the rod and the flywheel is of an offset distance, R_p , from the flywheel center. The wall of the expansion chamber is maintained at a higher temperature (T_H) and that of the compression chamber is at a lower temperature (T_L). The displacer of the engine is designed to move the gas medium back-and-forth between the two chambers. With the cam-drive mechanism, the displacer is able to move with a phase angle (Φ) ahead of the piston by adjusting the positions of the joints connecting the rods and the flywheel. The piston delivers the output power of the engine by flywheel rotation. All connecting rods of the engine are assumed to be rigid bodies with the centers of gravity located in the centers of the rods. All the engine components may experience both rotational and translational movement while the engine is operating.

The displacement of the piston and displacer with the cam-drive mechanism are described as:

$$Y_p = \frac{1}{2}l_p + R_p \cos\left(\frac{\pi}{2} - \theta\right) + l_1 \cos(\beta_1) \quad (1)$$

and

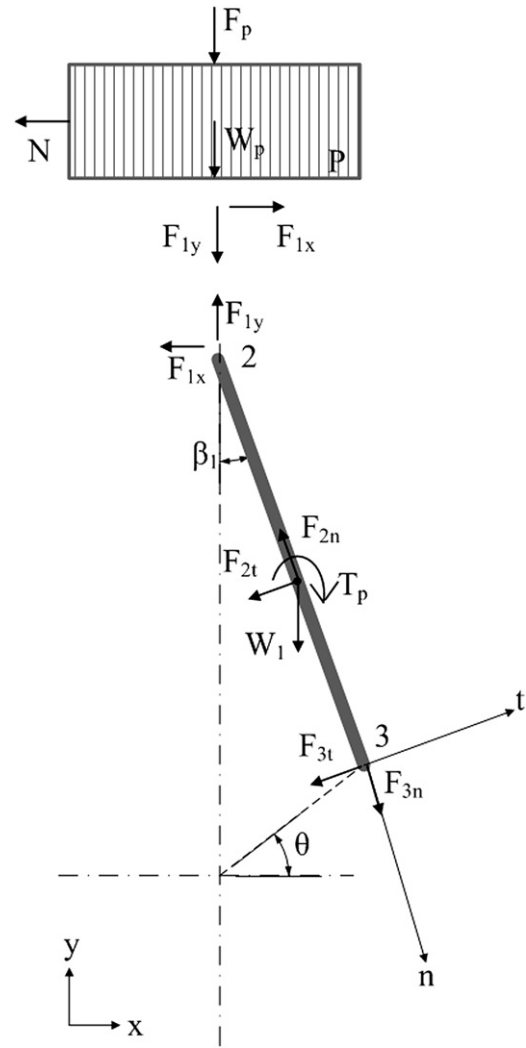


Fig. 2. Free-body diagram of piston and its connecting rod.

$$Y_d = l_d + l_{dt} + R_d \cos\left(\theta - \frac{\pi}{2} + \Phi\right) + l_2 \cos(\beta_2) \quad (2)$$

respectively, where

$$\beta_1 = \sin^{-1}\left(\frac{R_p}{l_1} \sin\left(\frac{\pi}{2} - \theta\right)\right) \quad (3)$$

and

$$\beta_2 = \sin^{-1}\left(\frac{R_d}{l_2} \sin\left(\theta - \frac{\pi}{2} + \Phi\right)\right) \quad (4)$$

The reciprocating velocities of the piston and displacer are obtained by differentiating Eqs. (1) and (2) with respect to time as

$$v_p = R_p \omega \sin\left(\frac{\pi}{2} - \theta\right) - l_1 \omega_{2-3} \sin(\beta_1) \quad (5)$$

and

$$v_d = -R_d \omega \sin\left(\theta - \frac{\pi}{2} + \Phi\right) - l_2 \omega_{5-6} \sin(\beta_2) \quad (6)$$

respectively, where $\omega (=d\theta/dt)$ is the angular speed of the flywheel; $\omega_{2-3} (=d\beta_1/dt)$ is the angular speed of link 2–3; and $\omega_{4-5} (=d\beta_2/dt)$ is the angular speed of link 5–6.

Furthermore, the accelerations of piston and displacer could be obtained by differentiating the velocity equations, and one has

$$a_p = R_p \alpha \sin\left(\frac{\pi}{2} - \theta\right) - R_p \omega^2 \cos\left(\frac{\pi}{2} - \theta\right) - l_1 \varepsilon_{2-3} \sin(\beta_1) - l_1 \omega_{2-3}^2 \cos(\beta_1) \quad (7)$$

$$a_d = -R_d \alpha \sin\left(\theta - \frac{\pi}{2} + \Phi\right) - R_d \omega^2 \cos\left(\theta - \frac{\pi}{2} + \Phi\right) - l_2 \varepsilon_{5-6} \sin(\beta_2) - l_2 \omega_{5-6}^2 \cos(\beta_2) \quad (8)$$

where α ($=d\omega/dt$) is the angular acceleration of the flywheel; ε_{2-3} ($=d\omega_{2-3}/dt$) is the angular acceleration of link 2–3; and ε_{5-6} ($=d\omega_{5-6}/dt$) is the angular acceleration of link 5–6.

2.1. Piston (1) and its connecting rod (link 2–3)

The dynamic simulation of all the parts of the mechanism is analyzed in this section. According to Fig. 2, for link 2–3, one has

$$l_1 \sin(\beta_1) = R_p \sin\left(\frac{\pi}{2} - \theta\right) \quad (9)$$

Differentiating Eq. (9) with respect to time yields

$$l_1 \cos(\beta_1) \frac{d\beta_1}{dt} = -R_p \omega \cos\left(\frac{\pi}{2} - \theta\right) \quad (10)$$

The angular velocity of the connecting rod 2–3 is then obtained as

$$\omega_{2-3} = -R_p \omega \cos\left(\frac{\pi}{2} - \theta\right) / l_1 \cos(\beta_1) \quad (11)$$

Angular acceleration of link 2–3 is obtained by further differentiating Eq. (11) with respect to time as:

$$\varepsilon_{2-3} = -\frac{R_p}{l_1} \cos(\beta_1) \left[\alpha \cos\left(\frac{\pi}{2} - \theta\right) + \omega^2 \sin\left(\frac{\pi}{2} - \theta\right) - \omega \omega_{2-3} \tan(\beta_1) \cos\left(\frac{\pi}{2} - \theta\right) \right] \quad (12)$$

As the above kinematic equations of the piston and rod 2–3 are derived, the dynamic equations are carried out with respect to the moving frame (n – t plane), and the torque balance equation can be written in terms of the crankpin axis as shown in Fig. 2. That is,

Force balance in n -direction:

$$F_{2n} - W_1 \cos(\beta_1) + F_{3n} - F_{1x} \sin(\beta_1) - F_{1y} \cos(\beta_1) = 0 \quad (13)$$

Force balance in t -direction:

$$F_{2t} - W_1 \sin(\beta_1) - F_{3t} + F_{1y} \sin(\beta_1) - F_{1x} \cos(\beta_1) = 0 \quad (14)$$

Torque balance with respect to joint 3:

$$-T_p + \frac{1}{2} F_{2p} l_1 + \frac{1}{2} W_1 l_1 \sin(\beta_1) + F_{1x} l_1 \sin(\beta_1) - F_{1y} l_1 \cos(\beta_1) = 0 \quad (15)$$

where

$$F_p = \pi r_2^2 (P_c - P_{atm}) \quad (16)$$

$$W_p = m_p g \quad (17)$$

$$F_{1y} = -F_p - W_p + m_p a_p \quad (18)$$

$$F_{2n} = \frac{1}{2} l_1 m_{2-3} \omega_{2-3}^2 \quad (19)$$

$$F_{2t} = \frac{1}{2} l_1 m_{2-3} \omega_{2-3}^2 \quad (20)$$

$$W_1 = m_{2-3} (g - a_p) \quad (21)$$

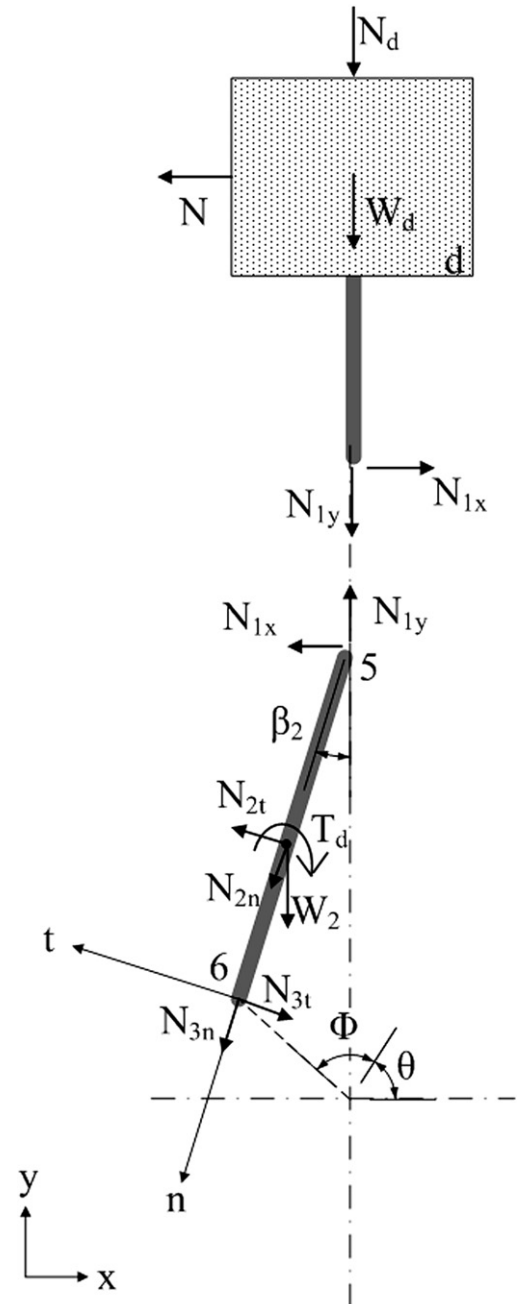


Fig. 3. Free-body diagram of displacer and its connecting rod.

$$T_p = l_{2-3}\varepsilon_{2-3} \quad (22)$$

Note that F_p is the periodic gas force caused by the pressure in the compression chamber which can be determined from the thermodynamic model [23], W_p is the weight of the piston assembly, and W_1 is the weight of the rod 2–3 plus the inertia effects caused by the piston's reciprocating movement. Equations (13)–(15) could be solved for the three unknown forces: F_{1x} , F_{3n} , and F_{3t} .

The tangential force exerted by the piston on the flywheel can then be written as

$$F_{tin} = -F_{3n}\sin\left(\frac{\pi}{2} - \theta + \beta_1\right) + F_{3t}\cos\left(\frac{\pi}{2} - \theta + \beta_1\right) \quad (23)$$

2.2. Displacer (4) and its connecting rod (link 5–6)

The free-body diagram of the displacer and its connecting rod 5–6 are plotted in Fig. 3. For the displacer and its connecting rod, one has

$$l_2\sin(\beta_2) = R_d\sin\left(\theta - \frac{\pi}{2} + \Phi\right) \quad (24)$$

Differentiating Eq. (24) with respect to time yields

$$l_2\cos(\beta_2)\frac{d\beta_2}{dt} = R_d\omega\cos\left(\theta - \frac{\pi}{2} + \Phi\right) \quad (25)$$

The angular velocity of the connecting rod can be given as

$$\omega_{5-6} = R_d\omega\cos\left(\theta - \frac{\pi}{2} + \Phi\right) / l_2\cos(\beta_2) \quad (26)$$

The angular acceleration of the connecting rod is

$$\varepsilon_{5-6} = \frac{R_d}{l_2}\cos(\beta_2)\left[\alpha\cos\left(\theta - \frac{\pi}{2} + \Phi\right) - \omega^2\sin\left(\theta - \frac{\pi}{2} + \Phi\right) - \omega\omega_{5-6}\tan(\beta_2)\cos\left(\theta - \frac{\pi}{2} + \Phi\right)\right] \quad (27)$$

Then, the force and torque balance equations for the displacer and its connecting rod are written with respect to crankpin axis as shown in Fig. 3 as follows:

Force balance in n -direction:

$$-N_{2n} - W_2\cos(\beta_2) + N_{3n} + N_{1x}\sin(\beta_2) - N_{1y}\cos(\beta_2) = 0 \quad (28)$$

Force balance in t -direction:

$$-N_{2t} + W_2\sin(\beta_2) - N_{3t} + N_{1y}\sin(\beta_2) + N_{1x}\cos(\beta_2) = 0 \quad (29)$$

Torque balance with respect to joint 3:

$$-T_d + \frac{1}{2}N_{2t}l_2 - \frac{1}{2}W_2l_2\sin(\beta_2) - N_{1y}l_2\sin(\beta_2) - N_{1x}l_2\cos(\beta_2) = 0 \quad (30)$$

where

$$F_d = \pi r_1^2(P_e - P_c) \quad (31)$$

$$W_d = m_d g \quad (32)$$

$$N_{1y} = -F_d - W_d + m_d a_d \quad (33)$$

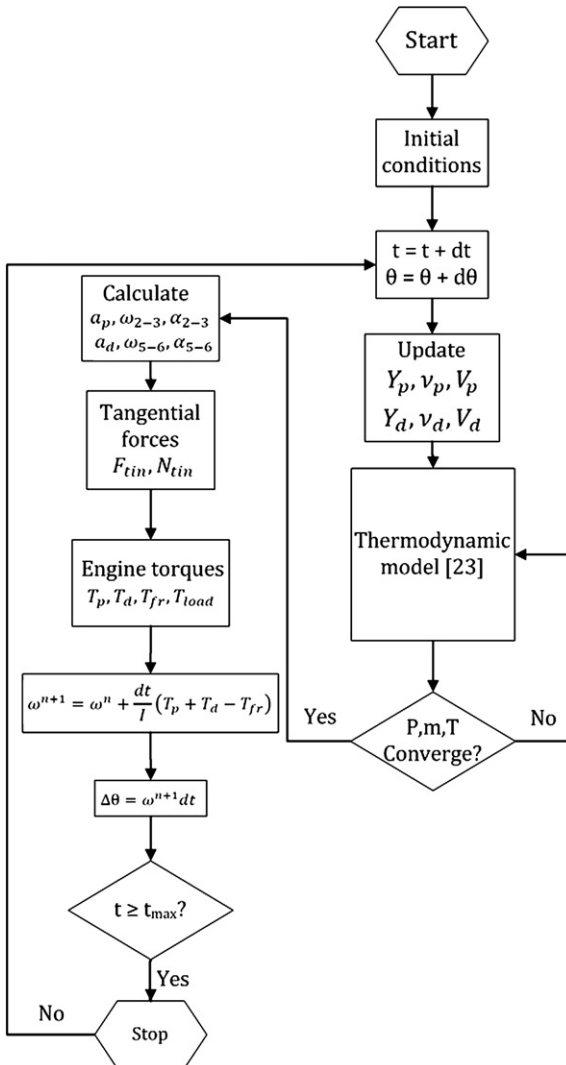


Fig. 4. Flow chart of the computational process.

Table 1

Geometrical variables of the baseline case.

Variable	Value	Variable	Value
R_p (m)	0.0095	ω_i (rpm)	1500
R_d (m)	0.0067	P_i (kPa)	101.3
l_1 (m)	0.079	T_H (K)	1300
l_2 (m)	0.055	T_c (K)	300
l_p (m)	0.0389	Φ (degree)	70
l_d (m)	0.045	dt (s)	0.000001
r_1 (m)	0.0095	ε_{eff}	0.75
r_2 (m)	0.0099	$R_{e,i}$ ($^{\circ}\text{C}/\text{kW}$)	40118.312
G (m)	0.0004	$R_{e,i}$ ($^{\circ}\text{C}/\text{kW}$)	40689
l_{dt} (m)	0.055	Gas medium	Air
l_t (m)	0.1724		
m_{2-3} (kg)	0.002187		
m_{5-6} (kg)	0.001563		
m_p (kg)	0.014013		
m_d (kg)	0.013292		
I ($\text{kg}\cdot\text{m}^2/\text{s}^2$)	0.0008		

Table 2

Operating conditions and the results of steady-state rotational speed, output power, and thermal efficiency of studied cases.

Case	ω_i (rpm)	P (kPa)	T_H (K)	Φ (degree)	ε_{eff}	I (kg·m ²)	G (m)	R_p (m)	l_d (m)	ω_{ave} (rpm)	W (W)	η
1	1500	101.3	1000	70	0.75	0.0008	0.0004	0.0067	0.045	899.63	7.9089	0.3580
2–1	1000	101.3	1000	70	0.75	0.0008	0.0004	0.0067	0.045	899.10	7.9087	0.3581
2–2	500	101.3	1000	70	0.75	0.0008	0.0004	0.0067	0.045	898.18	7.9093	0.3581
3–1	1500	101.3	1000	70	0.75	0.0001985	0.0004	0.0067	0.045	887.01	7.7958	0.3567
3–2	1500	101.3	1000	70	0.75	0.00008	0.0004	0.0067	0.045	849.27	7.4820	0.3521
3–3	1500	101.3	1000	70	0.75	0.004	0.0004	0.0067	0.045	1210.24	7.7870	0.3324
3–4	1500	101.3	1000	70	0.75	0.008	0.0004	0.0067	0.045	1335.44	7.6209	0.3203
4–1	1500	202.6	1000	70	0.75	0.0008	0.0004	0.0067	0.045	996.34	8.9492	0.3548
4–2	1500	303.9	1000	70	0.75	0.0008	0.0004	0.0067	0.045	1026.66	9.2891	0.3532
5–1	1500	101.3	800	70	0.75	0.0008	0.0004	0.0067	0.045	631.76	5.2236	0.3624
5–2	1500	101.3	1300	70	0.75	0.0008	0.0004	0.0067	0.045	1247.30	11.8237	0.3512
6–1	1500	101.3	1000	40	0.75	0.0008	0.0004	0.0067	0.045	933.56	6.6530	0.2938
6–2	1500	101.3	1000	50	0.75	0.0008	0.0004	0.0067	0.045	966.56	7.5149	0.3289
6–3	1500	101.3	1000	60	0.75	0.0008	0.0004	0.0067	0.045	944.66	7.8983	0.3496
6–4	1500	101.3	1000	80	0.75	0.0008	0.0004	0.0067	0.045	845.88	7.6635	0.3569
7–1	1500	101.3	1000	70	0.76	0.0008	0.0003	0.0067	0.045	843.41	7.2848	0.3404
7–2	1500	101.3	1000	70	0.72	0.0008	0.0005	0.0067	0.045	818.28	7.0795	0.3174
7–3	1500	101.3	1000	70	0.69	0.0008	0.0006	0.0067	0.045	726.15	6.1572	0.2761
8–1	1500	101.3	1000	70	0.75	0.0008	0.0004	0.0070	0.043	906.85	7.9903	0.3565
8–2	1500	101.3	1000	70	0.75	0.0008	0.0004	0.0070	0.047	960.88	8.5622	0.3915
9–1	1500	101.3	1000	70	0.75	0.0008	0.0004	0.0070	0.045	935.27	8.2887	0.3743
9–2	1500	101.3	1000	70	0.75	0.0008	0.0004	0.0064	0.045	862.90	7.5258	0.3419

$$N_{2n} = \frac{1}{2} l_2 m_{5-6} \omega_{5-6}^2 \quad (34)$$

$$N_{2t} = \frac{1}{2} l_2 m_{5-6} \varepsilon_{5-6} \quad (35)$$

$$W_2 = m_{5-6} (g - a_d) \quad (36)$$

$$T_d = I_{5-6} \varepsilon_{5-6} \quad (37)$$

In the above equations, F_d is the gas force caused by the pressure difference between expansion and compression chambers, which is calculated from the thermodynamic model [23], W_d is the weight of the displacer assembly, T_d is the torque on the center of gravity of the rod, and W_2 is the weight of the rod 5–6 plus the inertia force caused by the displacer's reciprocating movement. Equations (28)–(30) are solved for three unknown forces: N_{1x} , N_{3n} , N_{3t} . The tangential force exerted by the displacer on the flywheel can then be calculated as

$$N_{\text{tin}} = N_{3n} \sin\left(\theta - \frac{\pi}{2} + \Phi + \beta_2\right) - N_{3t} \cos\left(\theta - \frac{\pi}{2} + \Phi + \beta_2\right) \quad (38)$$

2.3. Equation of rotational motion of flywheel

It is assumed that the flywheel dominates the moment of inertia of the engine; therefore, the rotational speed of the engine can be determined by dealing with the rotational motion of the flywheel. The tangential forces, F_{tin} and N_{tin} , calculated by Eqs. (23) and (38), respectively, are introduced to determine the gas pressure torques exerted on the flywheel as:

$$T_p = F_{\text{tin}} R_p \quad (39)$$

$$T_d = N_{\text{tin}} R_d \quad (40)$$

On the other hand, the friction torque in the engine is also taken into account in this study. Note that a number of existing studies

[26,27] dealt with the friction model of the internal combustion engines. However, these friction models cannot be used for the beta-type Stirling engines whose structure is different from the internal combustion engines. In a recent report, Cheng and Yang [28] proposed an empirical correlation of the friction work loss in terms of the angular speed of a beta-type Stirling engine. According to the observation of [28], the friction torque is assumed to be proportional to the mechanical loss of the engine as

$$T_{\text{fr}} = c_{\text{fr}} \omega \quad (41)$$

where as a test case, the friction factor (c_{fr}) can be derived from the empirical correlation for the engine designed by Cheng and Yang [28]

$$c_{\text{fr}} = 2.98 \times 10^{-8} + 1.02 \times 10^{-4} \omega^{-1} \quad (42)$$

It is noticed that the coefficients of Eq. (42) are dependent on individual Stirling engine in use. A universal correlation for the friction factor suitable for all different types of Stirling engines is not realistic. For other type of Stirling engines, the form of friction model could even be altered dramatically.

The conservation of angular momentum is applied to yield the equation of rotational motion as

$$I d\omega/dt = T_p + T_d - T_{\text{fr}} - T_{\text{load}} \quad (43)$$

where I is the total mass moment of inertia of the engine's flywheel and T_{load} represents the external load of the engine. Eq. (43) can be integrated numerically with respect to time by means of the fourth-order Runge–Kutta method [29] to yield the transient variation in the rotational speed ω . Note that in the present dynamic model, the effects of friction and components' weights and mass inertia are included. Also, it is important to mention that the external load T_{load} is treated as a variable in the present model which can be specified in accordance with the practical loading condition. Here, the value of T_{load} is assigned to be zero so as to investigate the balance between the gas pressures and the friction force.

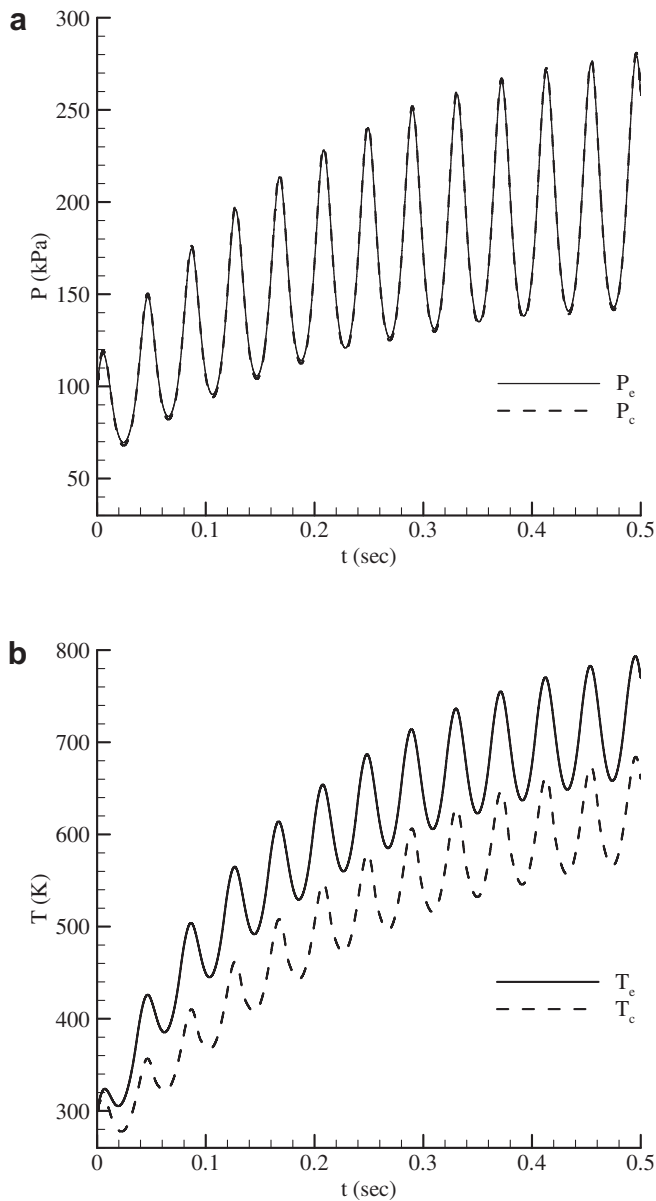


Fig. 5. Transient variations in pressures and temperatures in the expansion and the compression chambers, for baseline case (case 1).

3. Results and discussion

In the numerical simulation process, the engine is started from an initial rotational speed. The torques exerted by the flywheel of the engine at any time instant caused by the gas pressures in the chambers, the mass inertia, the friction force, and the external load are calculated. The instantaneous rotation speed of the engine is then determined by integration of the equation of rotational motion, Eq. (43), with respect to time. The obtained instantaneous rotation speed of the engine is introduced into the thermodynamic model to determine the instantaneous variations in pressure and other thermodynamic properties of the gas inside the chambers. The gas pressures are used to calculate the gas pressure torques, and the computation marches to the next time step. The transient variations in gas properties inside the engine chambers and the dynamic behavior of the engine mechanism are handled simultaneously via the coupling of the thermodynamic and dynamic models. Fig. 4 shows the flow chart of computation process. The

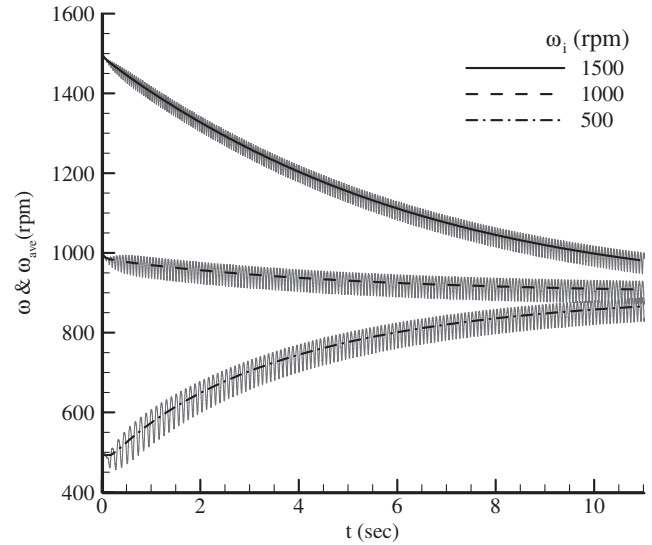


Fig. 6. Instantaneous and cycle-average rotational speeds from different initial speeds (cases 1, 2–1, and 2–2).

geometry specification and operating conditions of a baseline case are listed in Table 1. In the baseline case, initially the gas medium, air, is assumed to be 300 K in both chambers and the charged pressure is 101.3 kPa. The temperature of the wall of expansion chamber is maintained at T_H from the beginning of the simulation since the hot-start simulation is of major concern here. The value of T_H is changed in the parametric study whereas T_L is fixed at and 300 K. In addition to the baseline case, there are 20 more cases considered in a comprehensive parametric study. The operating conditions of these studied cases are given and numbered in Table 2, and the obtained results of the rotational speed, output work, and thermal efficiency are provided.

Fig. 5 shows the transient variations in pressures and temperatures of air in the expansion and compression chambers (P_e , P_c , T_e , and T_c). As shown in Fig. 5(a), the pressures in the expansion and the compression chambers are very close. It is reasonable because the engine is small so that the pressure within the cylinder of the engine is almost equal. When the regenerative channel gap is reduced, a larger pressure drop between the chambers may be

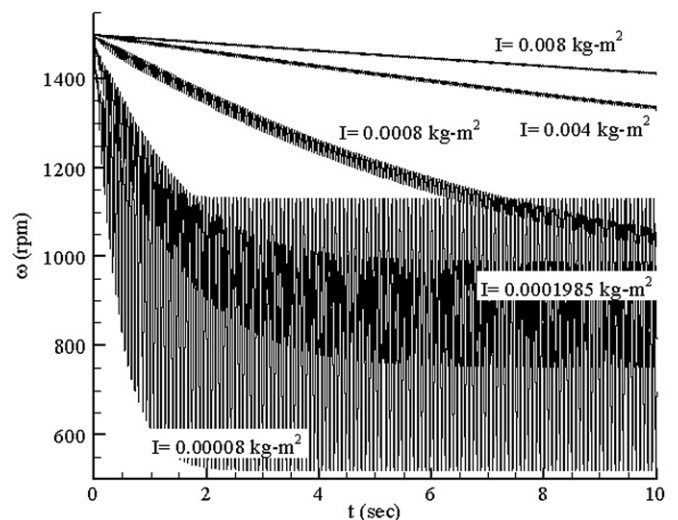


Fig. 7. Effects of moment inertia of flywheel on instantaneous and cycle-average rotational speeds (cases 1, 3–1, 3–2, 3–3, and 3–4).

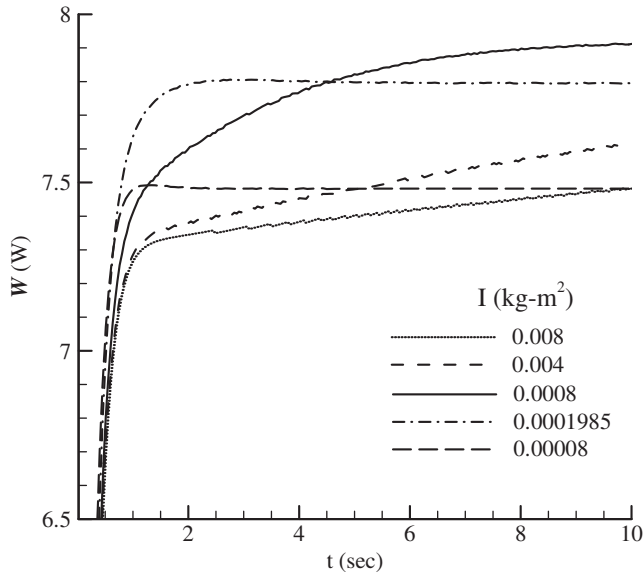


Fig. 8. Transient variations in cycle-average output power with different moment inertia of flywheel (cases 1, 3–1, 3–2, 3–3, and 3–4).

expected. In addition, the oscillating variation of the pressures reflects the periodic movement of the piston and displacer. The average pressure reaches a value of 200 kPa in about 0.5 s from start. The pressures inside the chambers lead to the gas force acting on the piston, displacer, and hence flywheel. In the second plot of this figure, the transient temperature variation in both chambers is depicted. The temperatures in the two chambers (T_e and T_c) are oscillatory and increased from 300 K to approximately 700 K and 600 K in the expansion and the compression chambers, respectively. Note that the difference between T_H and T_e in the steady regime is around 300 K and the difference between T_c and T_L is nearly of the same magnitude. The temperature differences ($T_H - T_e$ and $T_c - T_L$) are strongly related to the heats absorbed and rejected per cycle.

Fig. 6 shows the transient variation in the rotational speed of the engine starting from different initial rotational speeds ($\omega_i = 500, 1000, 1500$ rpm). The cycle-average rotational speed (ω_{ave}) is also calculated and is illustrated with black curves. It is observed that the initial rotational speed indeed has subtle influence on rotational speed in the start-up period; however, after the start-up period all different initial rotational speeds lead to the same final cycle-average rotational speed in the steady-state regime. This is reasonable because from Eq. (43) as in the steady-state regime, the time derivatives of the cycle-average quantities vanish, that is $d\omega_{ave}/dt = 0$. As a result, the steady-state cycle-average rotational speed finally reached should be

$$\omega_{ave} = (T_{p,ave} + T_{d,ave} - T_{load,ave})/C_{fr} \quad (44)$$

It implies that in the steady-state regime the cycle-average rotational speed (ω_s) is independent of the initial rotational speed. However, it is observed that only an initial rotational speed within a certain range can start the engine and make the engine reach the steady-state regime finally. The engine is brought to a stop if an improper initial rotational speed outside the range is chosen.

Fig. 7 conveys the effects of moment inertia of flywheel on the instantaneous and the cycle-average rotational speeds, for cases 1, 3–1, 3–2, 3–3, and 3–4. In this figure, the magnitude of the moment inertia of flywheel is varied from 0.00008 to 0.008 kg-m². It is found that the oscillating amplitude of the instantaneous

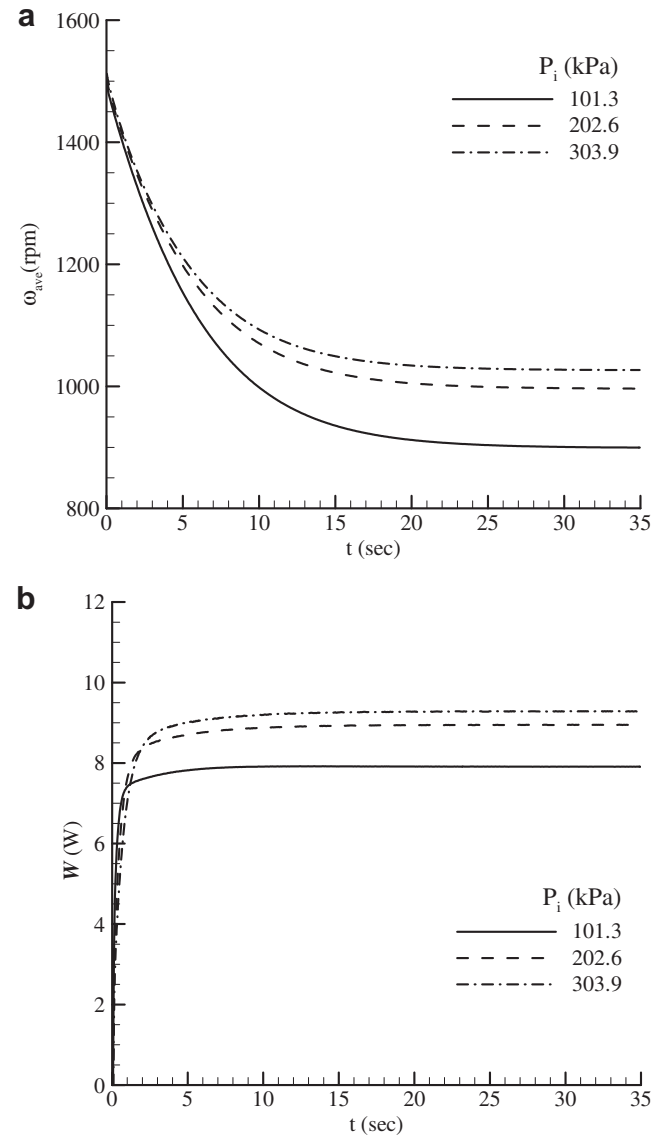


Fig. 9. Effects of initial charged pressure on cycle-average rotational speed and output power (cases 1, 4–1, and 4–2).

rotational speed is decreased by an increase of the moment of inertia of the flywheel. Meanwhile, for the case with a flywheel of larger moment inertia, it takes longer time for the engine to reach the steady-state regime. However, as the steady-state regime is reached, the cycle-average rotational speed (ω_s) approaches the same value regardless of the magnitude of moment inertia of flywheel.

In Fig. 8, the dependence of engine's cycle-average output power (W) on the moment of inertia of the flywheel is depicted for the same cases. In this figure, it is observed that the cycle-average output power in the steady-state regime attains a maximum of 7.91 W (and the thermal efficiency reaches a maximum of 0.358 as given in Table 2) when the value of I is equal to 0.0008 kg-m². This obviously implies that for any particular engine design, there exists an optimal value for the moment of inertia of the flywheel that should be determined in order to maximize the output power and the thermal efficiency.

Fig. 9 displays the effects of initial charged pressure on the cycle-average rotational speed and the cycle-average output power of the engine, for cases 1, 4–1, and 4–2. It is found that

both the cycle-average rotational speed and the cycle-average output power increase with the initial charged pressure. A higher initial charged pressure means a larger mass of air being charged in the cylinder of the engine. With more gas medium in the engine, the output power is expected to increase. The initial charged pressure is assigned to be 101.3, 202.6, and 303.9 kPa here. In Fig. 9 and Table 2, it is found that the power out achieves a maximum of 9.2891 W at $P = 303.9$ kPa. However, based on the thermal efficiency data shown in Table 2 for cases 1, 4–1 and 4–2, it is observed that an increase in initial charged pressure may lead to a decrease in the thermal efficiency. The reason for the decrease in the thermal efficiency accompanying the increase in the charged pressure is attributed to the higher heat input required for the engines containing more mass of air. An increase in mass of air elevates the thermal inertia of the gas medium; therefore, higher heat input is required to heat up the gas medium. In consequence, elevating the charged pressure of the engine could increase the engine's output power but might reduce the thermal efficiency of the engine.

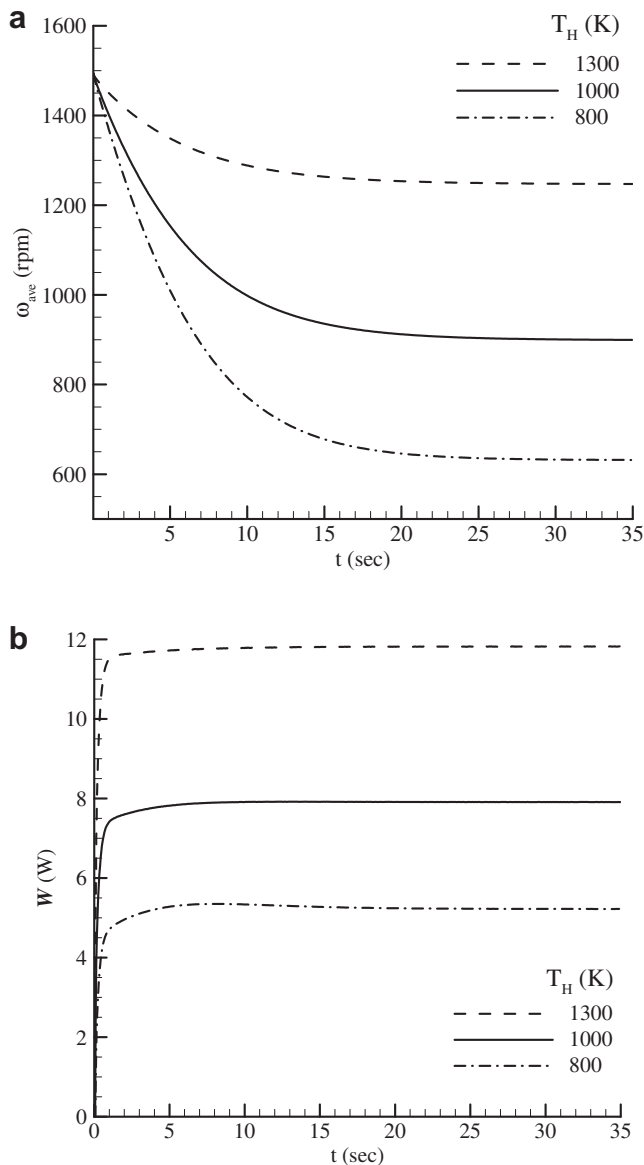


Fig. 10. Effects of heat source temperature on cycle-average rotational speed and output power (cases 1, 5–1, and 5–2).

Fig. 10 shows the heat source temperature effects on the performance of the engine. The heat source temperature is set at 800 K, 1000 K, or 1300 K. It is observed that the rotational speed and the output power are elevated by increasing the heat source temperature. However, the value of T_H is strictly restricted by the thermal expansion of the materials of the heating head. Since the temperatures of thermal reservoirs are given, the heat transfer to the chamber can be calculated in terms of the thermal resistance and the temperature difference between the fluid temperature and the heat source temperature. A higher heat source temperature means a higher heat transfer rate input to the expansion chamber.

Fig. 11 shows the effects of the phase angle between the displacements of the displacer and the piston on the cycle-average rotational speed and output power. The phase angle of the baseline case (case 1) is assigned to be 70 degrees. The simulation is made for four more phase angles, 50, 60, 80, and 82 degrees (cases 6–1, 6–2, 6–3, and 6–4) for comparison. This figure indicates that the optimal values of the phase angle leading to highest rotational speed or output power exist. According to the data provided in Fig. 11 and

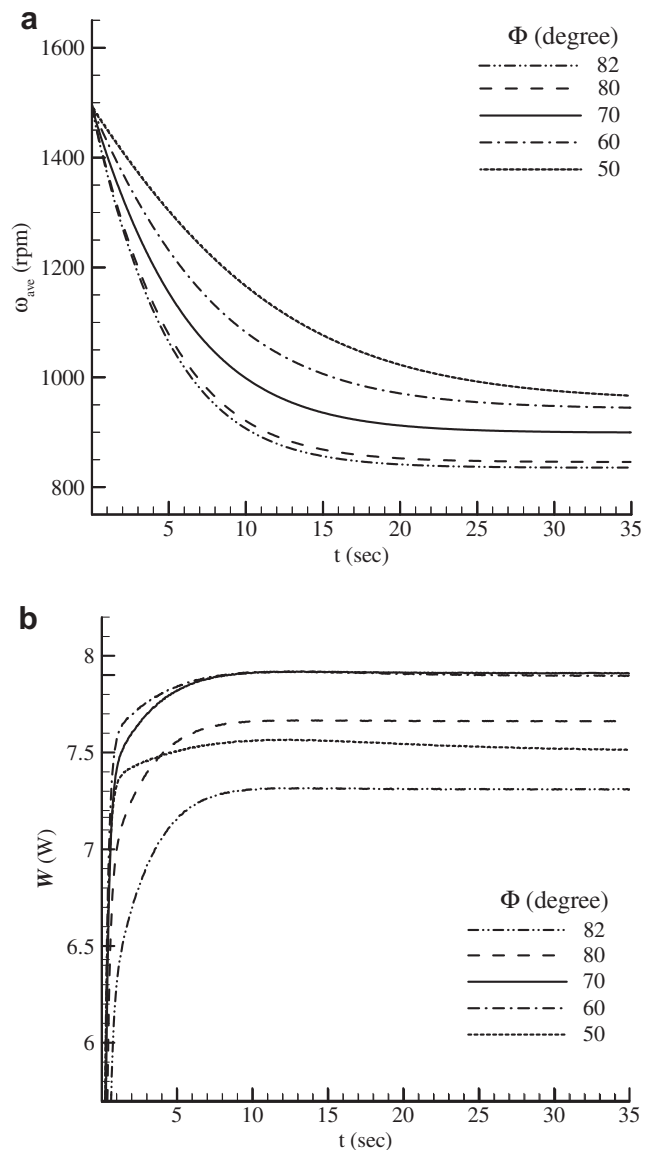


Fig. 11. Effects of phase angle on cycle-average rotational speed and output power (cases 1, 6–1, 6–2, 6–3, and 6–4).

Table 2, it is found that the case at $\phi = 50$ degrees leads to the highest cycle-average rotational speed in the steady-state regime, whereas the case at $\phi = 70$ degrees leads to the highest cycle-average output power and thermal efficiency. Note that the effects of the phase angle are rather involved. Typically, for the small-scale engines with efficient heat transfer between the gas medium and the walls of the expansion and the compression chambers, the optimal phase angle is set to be less than 90 degrees since the time required for heat transfer is shorter. On the contrary, for the larger-scale engine with relatively poor heat transfer, the optimal phase angle is in general higher than 90 degrees since the gas medium takes longer time for sufficient heat transfer. An adjustment in the phase angle should be made for different purposes.

Next, the effects of engine's geometrical parameters are investigated. The selected parameters are: regenerative gap size, length of the displacer, and the piston stroke.

Fig. 12 illustrates the subtle influence of the regenerative gap size. The regenerative gap size is varied from 0.0003 to 0.0006 m, corresponding to cases 1, 7–1, 7–2, and 7–3. As the gap size is decreased, significant increase in the shear friction loss is expected.

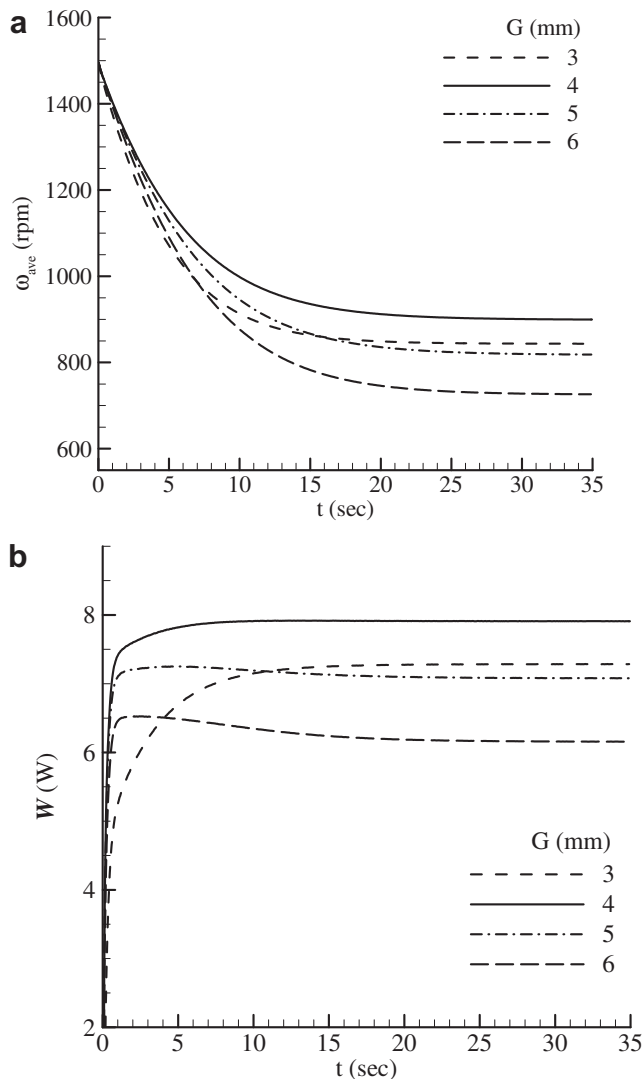


Fig. 12. Effects of regenerative gap size on cycle-average rotational speed and output power (cases 1, 7–1, 7–2, and 7–3).

As the gap size is increased, the regenerative effectiveness of the regenerative channels will be reduced. Therefore, there should be an optimal value for the gap size that leads to a peak value of the output power. For the particular case, the gap size of 0.0004 m exhibits the highest cycle-average rotational speed and the largest output power in the steady-state regime.

Fig. 13 shows the effects of the length of the displacer on the engine's rotational speed and output power. The rotational speed and the output power appear to increase with the displacer's length monotonically. As the stroke of the displacer is fixed, an increase in the length of the displacer means a reduction in the dead-zone space and an increase in the compression ratio. Therefore, the performance of the engine could be improved by increasing the displacer's length as long as the displacer would not collide with the walls of the cylinder and the piston.

Fig. 14 conveys the effects of piston stroke on the cycle-average rotational speed and output power, for cases 1, 9–1, and 9–2. The stroke of the piston is actually two times R_p . Therefore, in this figure R_p is used as the parameter varied from 0.0064 to 0.0070 m. It is found that both the cycle-average rotational speed and the

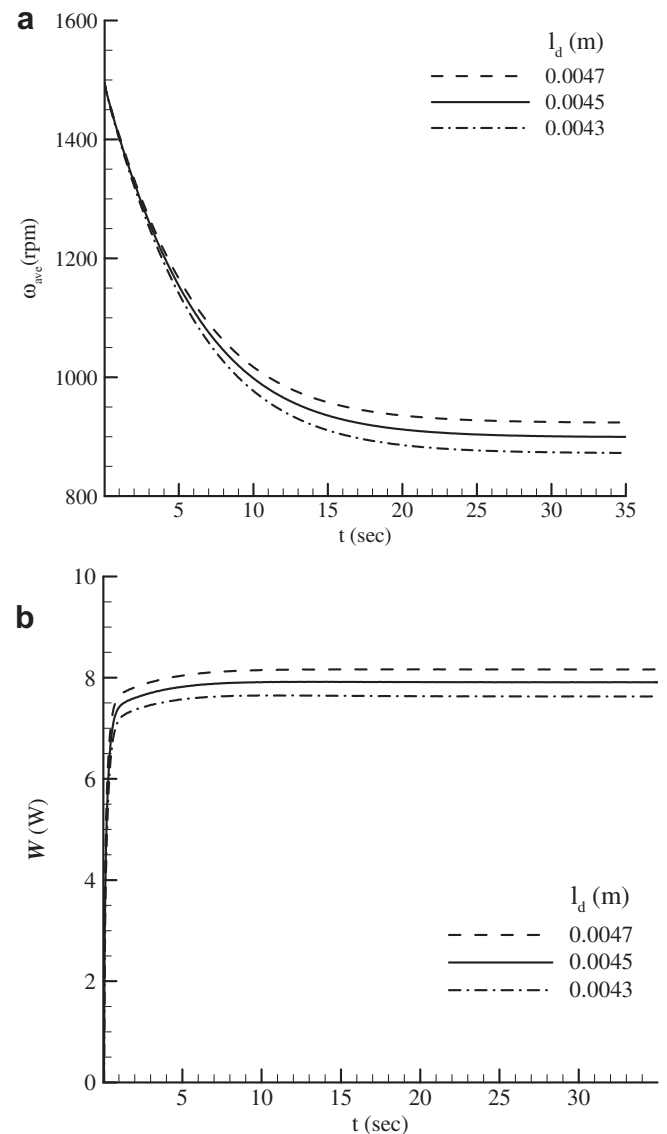


Fig. 13. Effects of displacer length on cycle-average rotational speed and output power (cases 1, 8–1, 8–2).

cycle-average output power increase with R_p . This is probably because a larger piston stroke leads to a higher compression ratio, which typically results in higher output power and thermal efficiency.

4. Concluding remarks

In this study, dynamic simulation of a beta-type Stirling engine with cam-drive mechanism has been attempted. A dynamic model associated with the cam-drive mechanism has been developed and combined with the thermodynamic model [23] to predict the transient behavior of the engine in the hot-start period. Based on the numerical simulation results, the following conclusions are drawn:

- (1) It is observed that the initial rotational speed and the moment inertia of flywheel exhibit subtle influence on rotational speed in the start-up period; however, after the period all different

initial rotational speeds and moment inertia of flywheel lead to the same final cycle-average rotational speed in the steady-state regime. In addition, for the particular cases, it is observed that the cycle-average output power and the thermal efficiency attain their respective maximums when the value of moment inertia of flywheel is equal to 0.0008 kg-m^2 .

- (2) Elevating the charged pressure of the engine could increase the engine's output power but might reduce the thermal efficiency of the engine.
- (3) Heat input to the expansion chamber increases with the heat source temperature. Therefore, it is observed that the rotational speed and the output power are elevated by increasing the heat source temperature. However, the value of T_H should be strictly restricted by the thermal expansion of the materials of the heating head.
- (4) Typically, for the small-scale engines with efficient heat transfer between the gas medium and the walls of the expansion and the compression chambers, the optimal phase angle is set to be less than 90 degrees since the time required for heat transfer is shorter. In the simulation, totally five angles, 50, 60, 70, 80, and 82 degrees are taken into consideration in comparison. It is found that the case at $\Phi = 50$ degrees leads to the highest cycle-average rotational speed in the steady-state regime, whereas the case at $\Phi = 70$ degrees leads to the highest cycle-average output power and thermal efficiency.
- (5) The effects of engine's geometrical parameters, including regenerative gap size, length of the displacer, and the piston stroke, are also investigated for the baseline case. Results show that the gap size of 0.0004 m exhibits the highest cycle-average rotational speed and the largest output power in the steady-state regime. In addition, since an increase in the length of the displacer or in the piston stroke leads to a reduction in the dead-zone space and an increase in the compression ratio, the performance of the engine could be improved by increasing the displacer's length or the piston stroke.

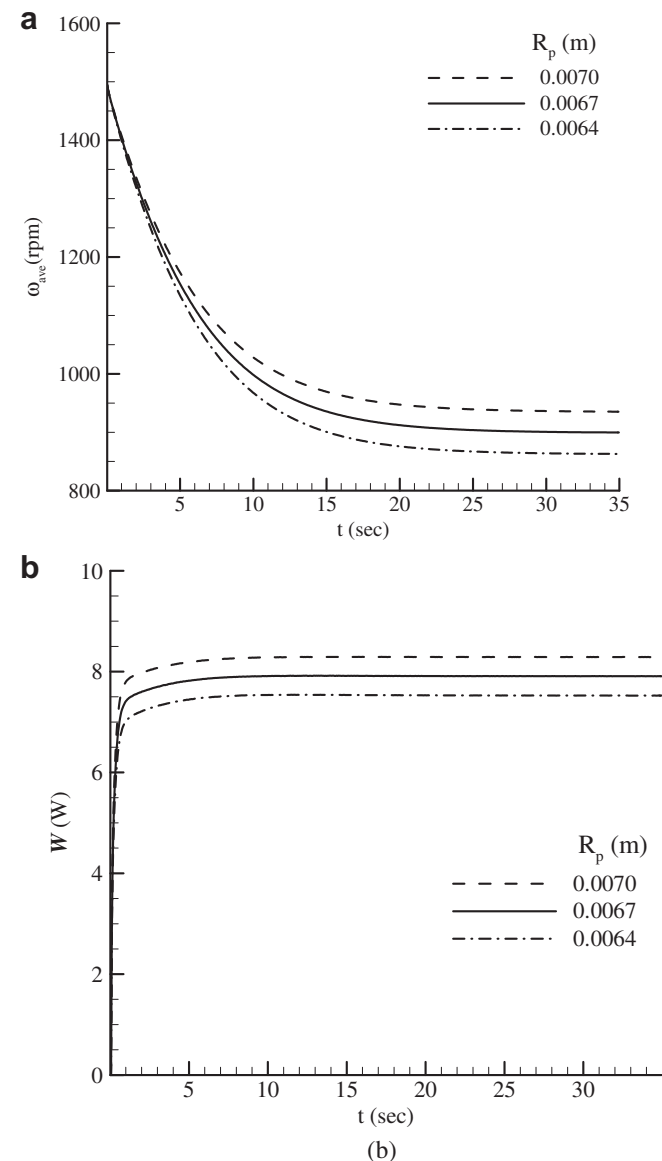


Fig. 14. Effects of piston stroke on cycle-average rotational speed and output power (cases 1, 9–1, and 9–2).

Acknowledgement

Financial support from the National Science Council, Taiwan, under grant NSC982C10 is greatly appreciated.

References

- [1] Thomas M, Peter H, Barry B, Bruce O, Wolfgang S, Vernon G, et al. Dish–Stirling system: an overview of development and status. *ASME J Sol Energy Eng* 2003;125:135–51.
- [2] SES SunCatcher, Available from: <<http://www.stirlingenergy.com>>.
- [3] Reinalter W, Ulmer S, Heller P, Rauch T, Gineste JM, Ferriere A, et al. Detailed performance analysis of a 10 kW dish/Stirling system. *ASME J Sol Energy Eng* 2008;130: 011013–1–6.
- [4] Mahkamov K. An axisymmetric computational fluid dynamics approach to the analysis of the working process of a solar Stirling engine. *ASME J Sol Energy Eng* 2006;128:45–53.
- [5] Liao CC. Analysis and evaluation of concentrating solar power system. Master thesis, Department of Energy of Mechatronics, National Central University, Taoyan, Taiwan, 2009.
- [6] Joachim G, Bernhard H, Stefan S, Markus S, Reiner B, Edgar T, et al. Solar concentrating systems using small mirror arrays. *ASME J Sol Energy Eng* 2010;132: 011003–1–4.
- [7] Yamaguchi H, Sawada N, Suzuki H, Ueda H, Zhang XR. Preliminary study on a solar water heater using supercritical carbon dioxide as working fluid. *ASME J Sol Energy Eng* 2010;132(1): 011010–1–6.
- [8] Redi S, Aglietti GS, Tatnall AR, Markvart T. An evaluation of a high altitude solar radiation platform. *ASME J Sol Energy Eng* 2010;132(1): 011004–1–8.
- [9] Walker G. *Stirling engines*. Oxford: Clarendon Press; 1980.
- [10] Berchowitz DM. The development of a 1 kW electrical output free piston Stirling engine alternator unit. In: *Proceedings of the IECEC*. Orlando, Florida; 1983. pp. 897–901.
- [11] Thombare DG, Verma SK. Technological development in the Stirling cycle engines. *Renew Sust Energy Rev* 2008;12(1):1–38.

- [12] Leonardo S, Pablo V, Jorge B. Design and construction of a Stirling engine prototype. *Int J Hydrogen Energy* 2008;33:3506–10.
- [13] Francois N, Alain F, Françoise B. Thermal model of a dish/Stirling systems. *Sol Energ* 2009;83:81–9.
- [14] Kongtragool B, Wongwises S. A review of solar-powered Stirling engines and low temperature differential Stirling engines. *Renew Sust Energ Rev* 2003;7: 131–54.
- [15] Schmidt G. Theorie der Lehmannschen calorischen maschine. *Zeit Des Ver-eines Deutsch Ing* 1871;15:1–12.
- [16] Finkelstein T. Thermodynamic analysis of Stirling engines. *J Spacecraft Rocket* 1967;4(9):1184–9.
- [17] Kongtragool B, Wongwises S. Thermodynamic analysis of a Stirling engine including dead volumes of hot space, cold space and regenerator. *Renew Energ* 2006;31(3):345–59.
- [18] Jones JD. Optimization of Stirling engine regenerator design. In: *Proceedings of the IECEC*; 1990. p. 359–65.
- [19] Mahkamov K. Design improvements to a biomass Stirling engine using mathematical analysis and 3D CFD modeling. *ASME J Energy Res Technol* 2006;128:203–15.
- [20] Urieli I, Berchowitz D. Stirling cycle engine analysis. Bristol: Adam Hilger; 1984.
- [21] Parlak N, Wagner A, Elsner M, Soyhan HS. Thermodynamic analysis of a gamma type Stirling engine in non-ideal adiabatic conditions. *Renew Energ* 2009;34(1):266–73.
- [22] Karabulut H, Aksoy F, Ozturk E. Thermodynamic analysis of a beta type Stirling engine with a displacer driving mechanism by means of a lever. *Renew Energ* 2009;34(1):202–8.
- [23] Cheng CH, Yu YJ. Numerical model for predicting thermodynamic cycle and thermal efficiency of a beta-type Stirling engine with rhombic-drive mechanism. *Renew Energ* 2010;35(11):2590–601.
- [24] Giakoumis EG, Rakopoulos CD. Simulation and analysis of a naturally aspirated IDI diesel engine under transient conditions comprising the effect of various dynamic and thermodynamic parameters. *Energy Convers Manage* 1998;39:465–84.
- [25] Giakoumis EG, Rakopoulos CD, Dimaratos AM. Study of crankshaft torsional deformation under steady-state and transient operation of turbocharged diesel engines. *J Multi-body Dyn* 2008;222(1):17–30.
- [26] Kikuchi T, Ito S, Nakayama Y. Piston friction analysis using a direct-injection single-cylinder gasoline engine. *Proc JSAE Annu Congr* 2001;115:1–4.
- [27] Nehme HK, Chalhoub NG, Henein NA. Effects of filtering the angular motion of the crankshaft on the estimation of the instantaneous engine friction torque. *J Sound Vib* 2000;236(5):881–94.
- [28] Cheng CH, Yang YJ. Design and performance test of a miniature Stirling generator. In: *Proceedings of the 26th National Conference of the CSME*, Tainan, Taiwan, Nov. 21–22, 2010.
- [29] Burden RL, Faires JD. Numerical analysis. 8th ed. Thomson Brooks/Cole; 2005.